Near-Optimal and Explicit Bell Inequality Violations

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Local realism?

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  - **Locality**: no faster than light influences.
  - **Realism**: values are determined before measurement.
- *[EPR'35]*: Quantum physics seems to violate local realism. Is it wrong or incomplete?
- *[Bell'64]*: Every local realistic theory must satisfy certain constraints (Bell Inequality).
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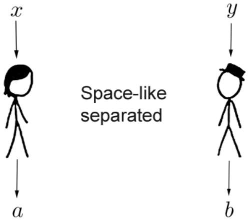
Non-local games



- Alice receives x and Bob receives y, where (x, y) are chosen from the distribution π.
  Alice outputs a and Bob outputs b.
- A *predicate* specifies winning outputs.
- *Goal*: maximize winning probability.
- Classical strategies: functions $A(x)$, $B(y)$.
  - The **classical value** $\omega(G)$ is the maximum winning probability over all classical strategies.
- Quantum strategies: shared *entangled* state;
  for each x measurement $\{A_a^x\}$; for each y $\{B_b^y\}$.
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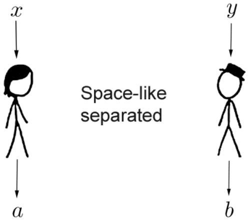

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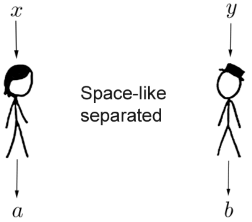

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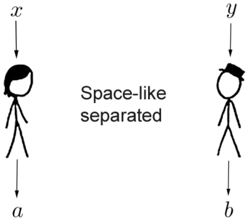

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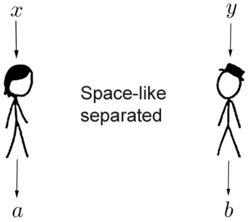

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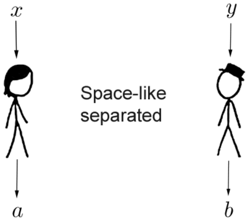

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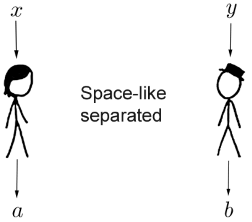

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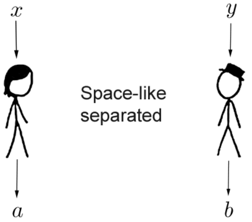

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Bell Inequality Violation

- A **Bell Inequality** is an upper bound on $\omega(G)$.
- **Violation**: $\omega^*(G)$ larger than $\omega(G)$.
  - Quantified by ratio $\frac{\omega^*(G)}{\omega(G)}$.
- CHSH *[Clauser, Horne, Shimony, Holt, 1969]*

  Classic example where $\frac{\omega_2^*(\text{CHSH})}{\omega(\text{CHSH})} \sim \frac{0.85}{0.75}$
- We want *large violations*!
  - Strong separation between quantum and classical worlds.
  - Typically easier to verify experimentally.

Study violation as a function of:

- Local dimension of the entangled state.
- Number of outputs.

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How large can the ratio $\frac{\omega_n^*(G)}{\omega(G)}$ be?

Upper Bounds:

- *[Junge, Palazuelos, Pérez-García, Villanueva, Wolf '09]*: with n-dimensional entanglement: $O(n)$.
- *[Junge, Palazuelos '10]*: with k possible outputs: $O(k)$.

Lower Bounds:

- *[Folklore]*: n^{ε} by parallel repetition of "magic square".
- *[Kempe, Regev, Toner '08]*: $n^{\varepsilon'}$ from Unique Games.
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- Variant of "Hidden Matching" from communication complexity. [Bar-Yossef, Jayram, Kerenidis, STOC'04].
- n outputs; entanglement dimension n.
- Violation of order $\sqrt{n}/\log n$.

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- From an example of integrality gap for Unique Games [Khot, Vishnoi, FOCS'05] and a Quantum Rounding technique [Kempe, Regev, Toner, FOCS'08]
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Our results

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What are the inputs?

$$x$$

$$1$$

$$0$$

$$1$$

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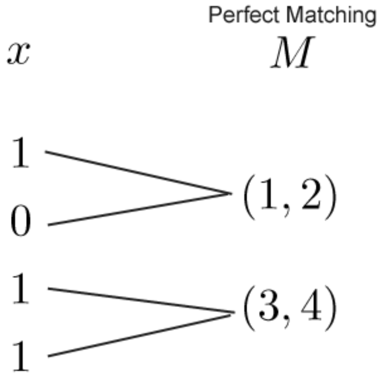

Hidden Matching *communication* game

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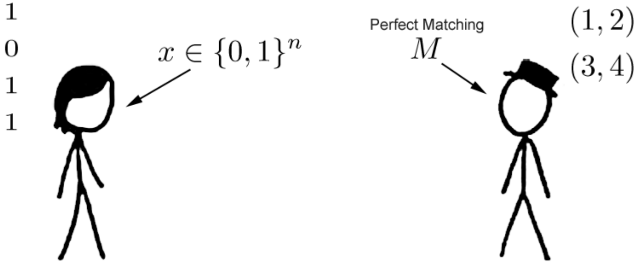

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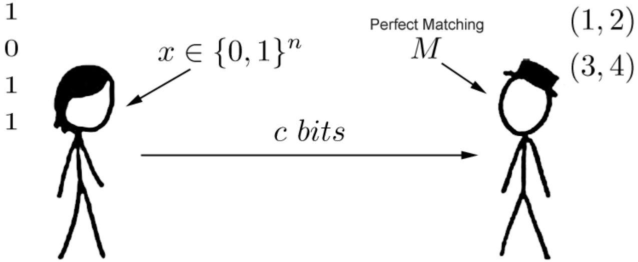

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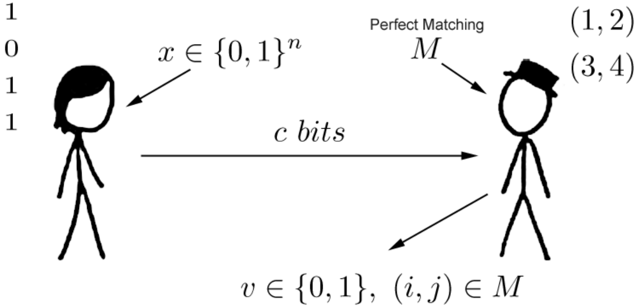

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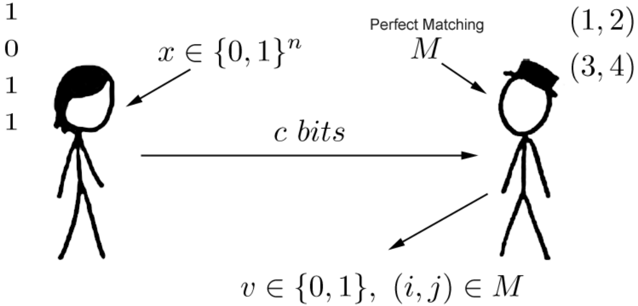

They win if $v = x_i \oplus x_j$.

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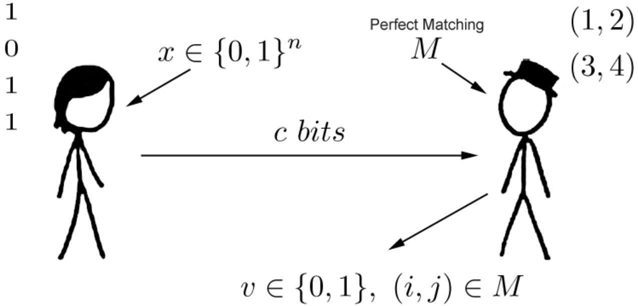

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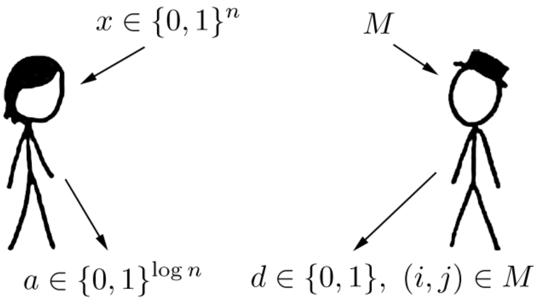
Thm: Classical winning probability is at most $\frac{1}{2} + O\left(\frac{c}{\sqrt{n}}\right)$ (*[BJK'04]* proved this for $c = \sqrt{n}$).

Hidden Matching *non-local* game



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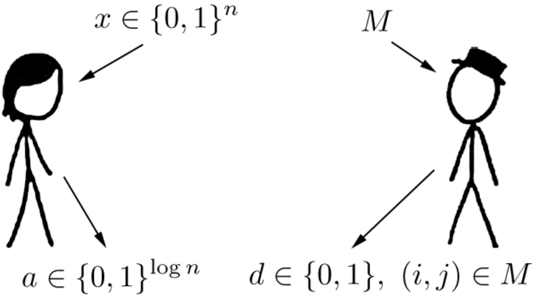

They win if $(a \cdot (i \oplus j)) \oplus d = x_i \oplus x_j$.

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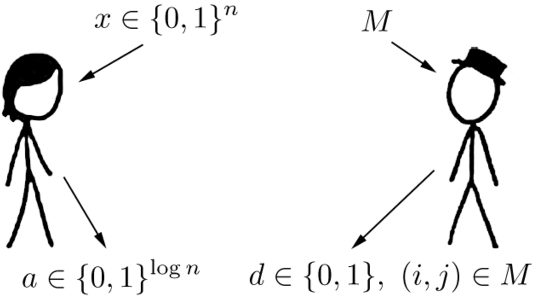

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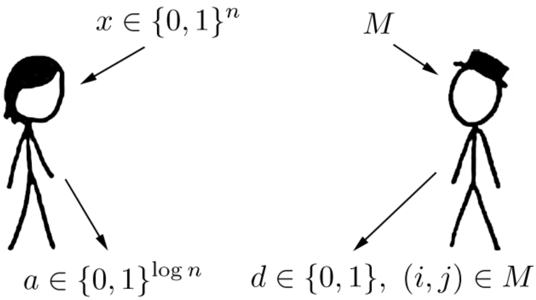

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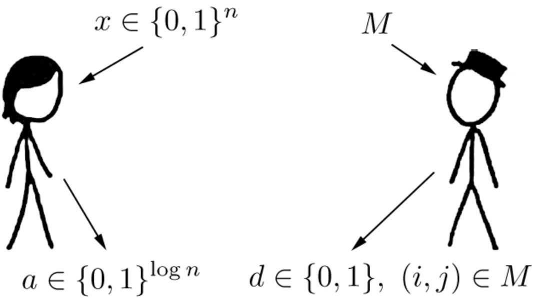

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Winning probability 1 with n-dimensional *entanglement*.
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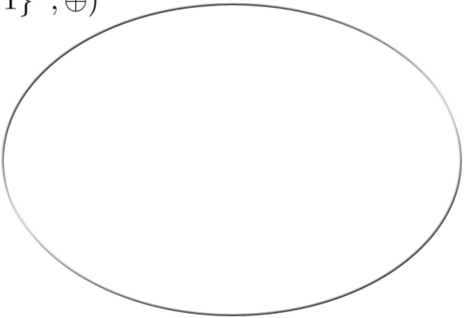
Khot-Vishnoi game

$G(\{0,1\}^n, \oplus)$



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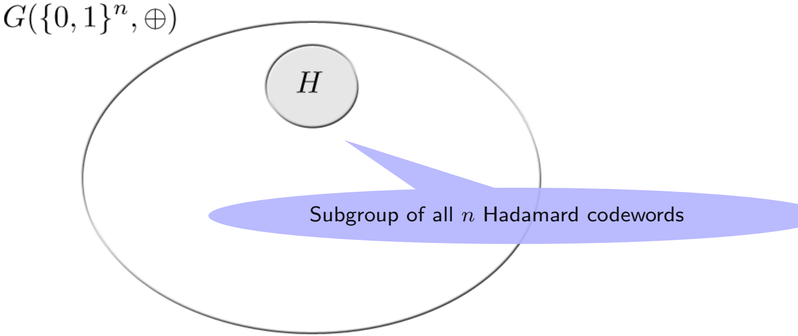

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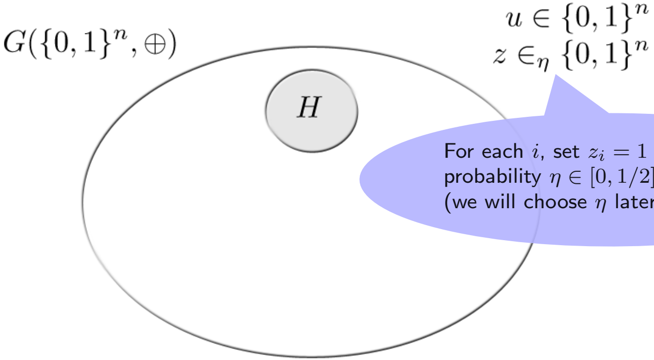
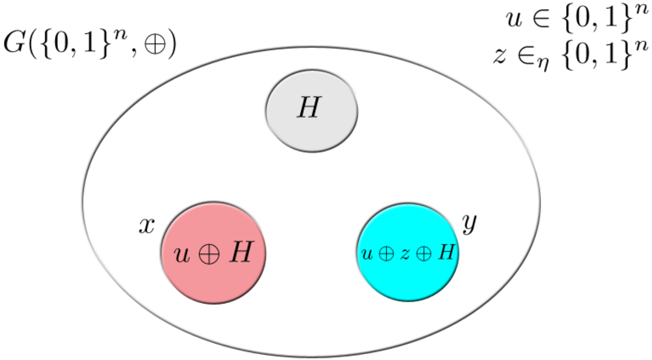


For each i, set $z_i = 1$ with probability $\eta \in [0, 1/2]$ (we will choose η later close to $\frac{1}{2}$)

Khot-Vishnoi game



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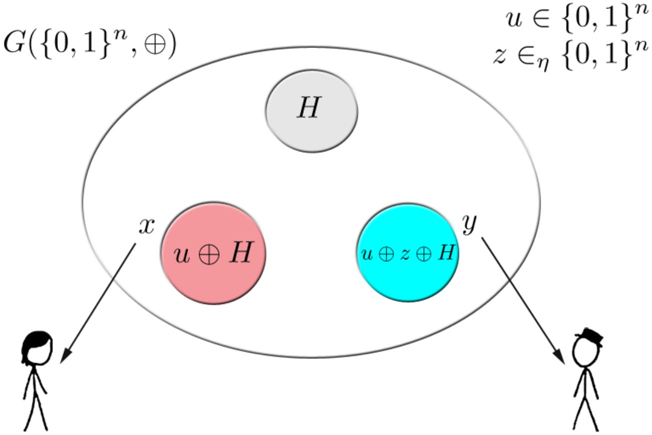

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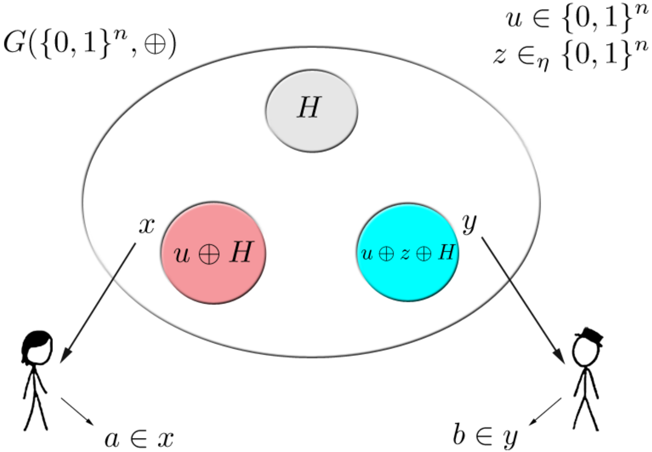

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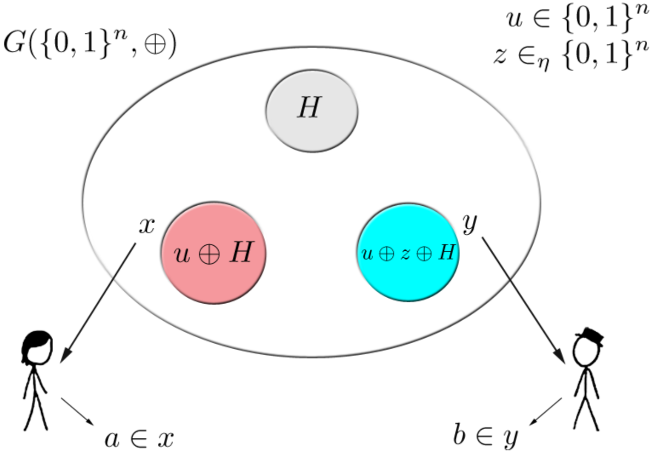

Winning condition: $a \oplus b = z$.

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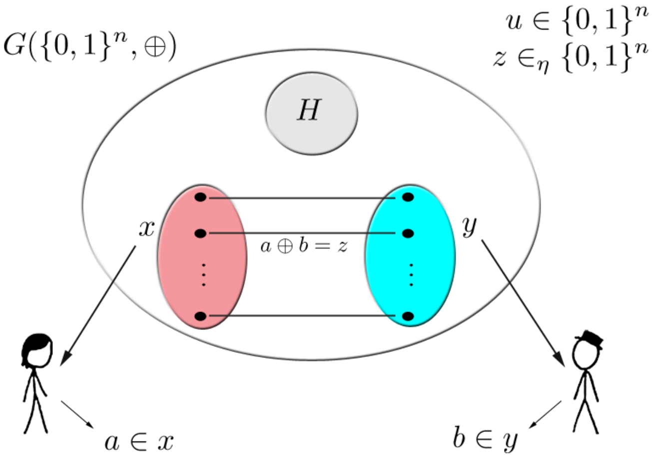

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Khot-Vishnoi - Quantum strategy

For any n and $\eta \in [0, 1/2]$, there exists a quantum strategy that wins with probability at least $(1 - 2\eta)^2$.

- For $a \in \{0,1\}^n$, define $|v^a\rangle = ((-1)^{a_i}/\sqrt{n})_{i \in [n]}$.
  - For all a, b, $\langle v^a, v^b \rangle = 1 - 2d(a,b)/n$.
  - The vectors $\{v^a \mid a \in x\}$ are an orthonormal basis of $\mathbb{R}^n$.
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Winning probability is at least $(1-2\eta)^2$.

- Probability to obtain a, b is $\frac{\langle v^a, v^b \rangle^2}{n}$.
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Khot-Vishnoi - Classical bound

Every classical strategy has winning probability $\leq 1/n^{\eta/(1-\eta)}$

- Fix strategy. Functions $A, B : \{0,1\}^n \to \{0,1\}$.
  - $A(u) = 1 \Leftrightarrow$ Alice's output on coset $u \oplus H$ is u.
  - $\mathbb{E}_u[A(u)] = 1/n$ (Alice chooses one element per coset)
  - Players win $\Leftrightarrow \sum_{h \in H} A(u \oplus h) B(u \oplus z \oplus h) = 1$.
- Winning probability is $\mathop{\mathbb{E}}_{u,z}[\sum_{h \in H} A(u \oplus h) B(u \oplus z \oplus h)]$

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Setting $\eta = \frac{1}{2} - \frac{1}{\log n}$

- Entangled value $(1-2\eta)^2 \sim \frac{1}{(\log n)^2}$
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