Quantum simulation of time-dependent Hamiltonians and the convenient illusion of Hilbert space

David Poulin

Département de Physique
Université de Sherbrooke

Joint work with: Angie Quarry, Rolando Somma, and Frank Verstraete

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Outline

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The simulations problem

Statement of the problem

- Input: A k-local Hamiltonian $H = \sum_{X \subset \{1,2,\ldots N\}} h_X$
  - $\|h_X\| \leq 1$, Hermitian
  - $h_X = 0$ for all $|X| > k$ (k-local)
- Output $\|\sigma U(t)|\psi\rangle\|^2$ for some simple state ψ and operator σ (E.g. $|\psi\rangle = |0\rangle$ and $\sigma = \sigma_1^z$)
  - Evolution operator $U(t) = \exp(-iHt)$

Why is this interesting

- Most modern questions in theoretical physics boil down to this
  - Hubbard model for high T_C superconductivity
  - Standard model for particle masses
  - Coulomb force for molecular binding energies
- Large fraction of the world's use of supercomputers

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... this was one of the original motivation to build one.

Main idea: Trotter-Lie-Suzuki

$$\exp(A+B) = \exp(A)\exp(B) + \mathcal{O}([A,B])$$

- Use iteratively to express ($\delta = t/M$)

$$e^{-iHt} = \left[e^{-iHt/M}\right]^M = \left[\prod_X e^{-ih_X\delta}\right]^M + \mathcal{O}(t \cdot poly(N)/M)$$

- Each term $e^{-ih_X\delta}$ is a k-body unitary: efficient by Solovay-Kitaev
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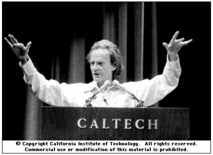

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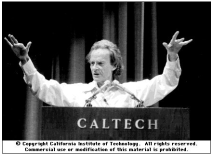

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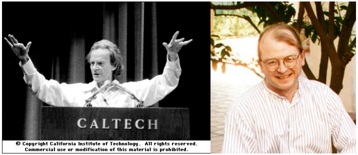

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- What if the Hamiltonian is time-dependent $H(t) = \sum_X h_X(t)$?
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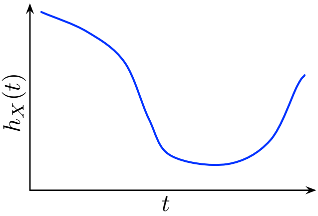

- Ignore the time-dependence on each time interval δ

$$U(t) \approx \ldots \times e^{-i\delta H(2\delta)} \times e^{-i\delta H(\delta)} \times e^{-i\delta H(0)}$$

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Complexity of the simulation depends on the smoothness of $H(t)$

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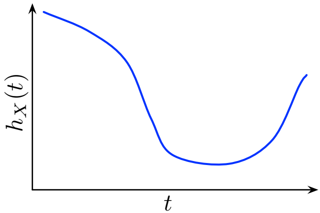

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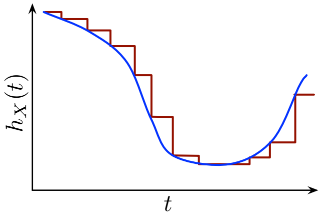

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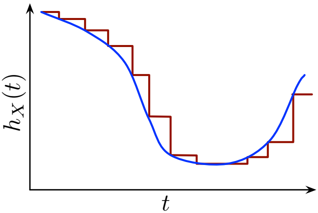

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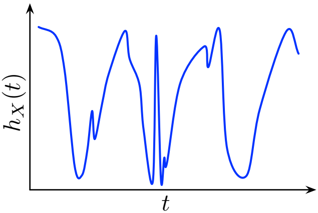

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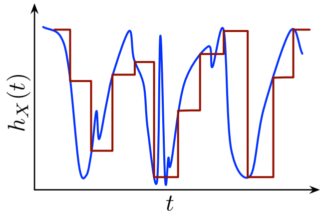

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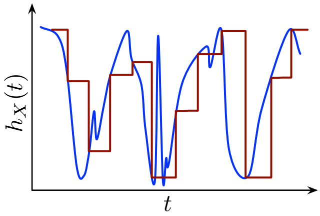

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Is this the best that can be done?

Wiebe, Berry, Høyer, and Sanders 2010 :

The $\{h_X\}$ are P-Λ-Smooth

$$\sup_{p\in\{1,2,\ldots,P\},t} \left[\sum_X \left\| \frac{d^p}{dt^p} h_X(t) \right\| \right]^{1/(p+1)} \leq \Lambda$$

Approximation $\|U(t) - \prod_{q=1}^{M} e^{-ih_{X_q}(t_q)\delta_q}\| \leq \epsilon$

Complexity $M \leq 3m\Lambda t k_0 \exp\left(k_0 2 \ln \frac{25}{3}\right)$ with $k_0 = \sqrt{\frac{1}{2}\log_{25/3}\frac{\Lambda\delta}{\epsilon}}$
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The complexity scales polynomially with the norm of the derivatives.

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1. Introduction
2. Decoupling principle
3. Quantum algorithm
4. Application: The hollowness of Hilbert space

Physical intuition

You don't need to understand quark confinement to predict the spectrum of the hydrogen atom, even though it's made of quarks.

- High-energy details are absorbed in a few effective parameters of the low-energy model
- Used in gadget hamiltonian perturbation theory for QMA

The dynamics of the system is largely insensitive to the high-energy

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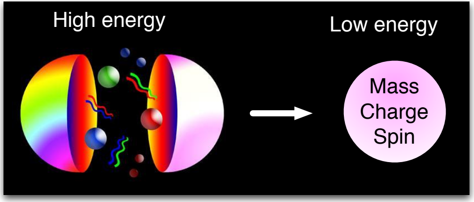

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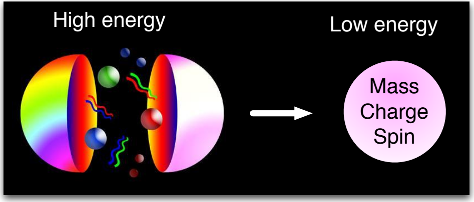

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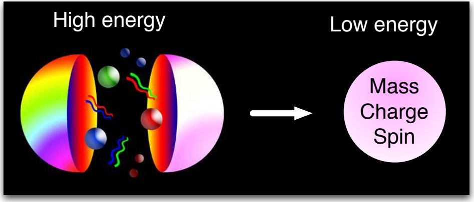

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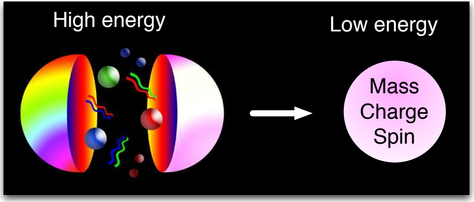
- Used in gadget hamiltonian perturbation theory for QMA

The dynamics of the system is largely insensitive to the high-energy

Physical intuition

You don't need to understand quark confinement to predict the spectrum of the hydrogen atom, even though it's made of quarks.



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Effective Hamiltonian

Frequency cutoff

- $H(t) = \int_0^\infty J(\omega)e^{i\omega t}d\omega$
- $\tilde{H}(t) = \int_0^\Gamma J(\omega)e^{i\omega t}d\omega$

- Frequency ω drives transition $E \to E + \omega$.
- If $\Gamma \gg \|H\|$, don't loose anything

Theorem

- Smooth cutoff $\tilde{H}(t) = \int \chi_\sigma(t-t')H(t')dt'$
- Evolutions $\|U(t) - \tilde{U}(t)\| \leq 2\|H\|^2 t\sqrt{\frac{2}{\pi}}\sigma$

Proof

- $X(t) = I - U^\dagger(t)\tilde{U}(t)$
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Randomized product formula

Time bins (exact)

$$\mathcal{T}e^{-i\int_0^T H(t)dt} = \mathcal{T}e^{-i\int_{T-\delta}^T H(t)dt} \ldots \mathcal{T}e^{-i\int_\delta^{2\delta} H(t)dt} \times \mathcal{T}e^{-i\int_0^\delta H(t)dt}$$

Remove time order (approximate)

$$\left\| \mathcal{T}e^{-i\int_0^\delta H(t)dt} - e^{-i\int_0^\delta H(t)dt} \right\| \leq 2\|H\|^2\delta^2, \text{ decoupling principle}$$

Monte Carlo integral (approximate)

For $t_j \in_R [0,\delta]$, $\left\| \frac{1}{\delta}\int_0^\delta H(t)dt - \frac{1}{m}\sum_{j=1}^m H(t_j) \right\| \leq \|H\| \frac{\delta}{\sqrt{m}}$, w.h.p.

Trotter decomposition (approximate)

$$\left\| e^{-i\frac{\delta}{m}\sum_{j=1}^m H(t_j)} - \prod_{j=1}^m e^{-i\frac{\delta}{m}H(t_j)} \right\| \leq \frac{\delta^2}{m}\|H\|^2$$

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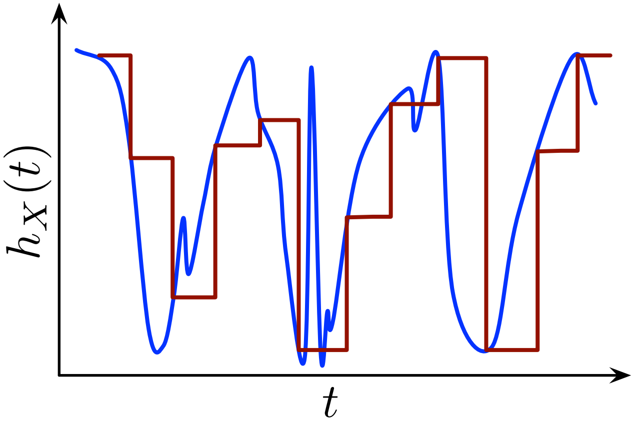

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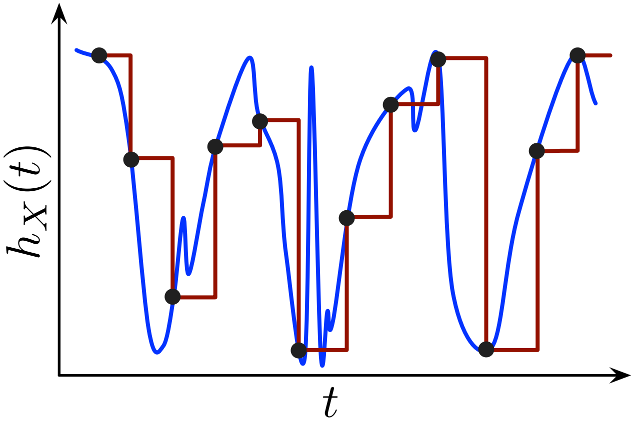

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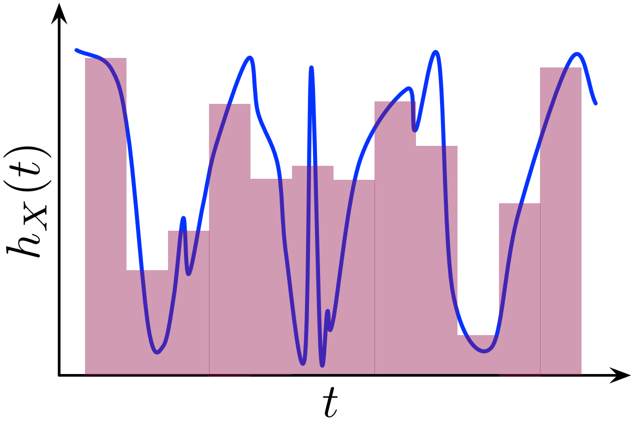

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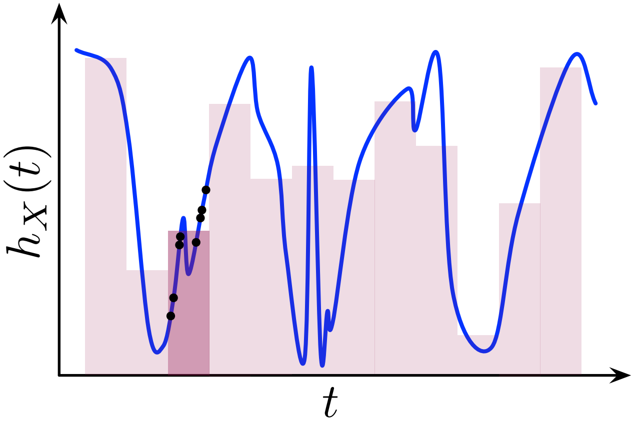

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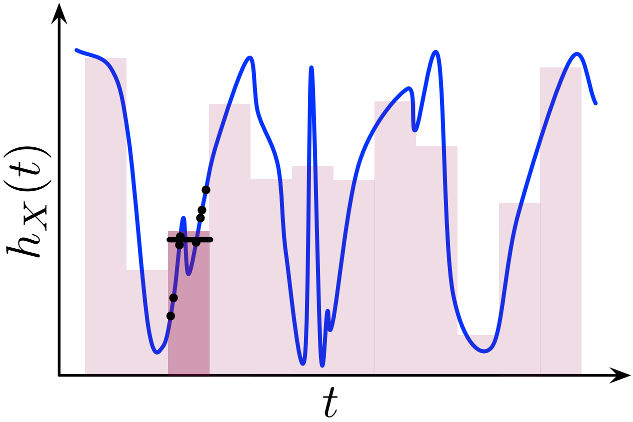

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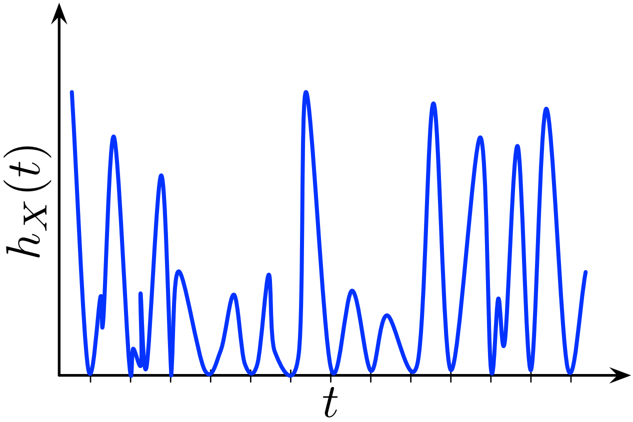

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Variational states

- Hilbert space is big.
- Physical systems appear to occupy a tiny sub-manifold:
  - Matrix product states and PEPS
  - Laughlin state for fractional quantum Hall liquids
  - BCS state for superconductivity
  - etc.

Proposed definition: physical state of quantum many-body system

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How many physical states?

- A direct counting argument appears difficult because $H(t)$ can vary arbitrarily wildly.

Corollary to our result

Vol(physical states) $\leq$ # poly-size quantum circuits $\times$ Vol(ϵ ball)

Number of quantum circuits

- $\mathcal{M} =$ finite gate set
- $K =$ number of qubits
- $\alpha =$ degree sim. poly

$$N_C \leq (|\mathcal{M}|K^2)^{K^\alpha}$$

Number of quantum states

- Size of $\mathcal{H}$ = Vol($2^{K+1} - 1$ sphere)
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- A direct counting argument appears difficult because $H(t)$ can vary arbitrarily wildly.

Corollary to our result

Vol(physical states) $\leq \#$ poly-size quantum circuits $\times$ Vol(ϵ ball)

Number of quantum circuits

- $\mathcal{M}$ = finite gate set
- K = number of qubits
- α = degree sim. poly

$$N_C \leq (|\mathcal{M}|K^2)^{K^\alpha}$$

Number of quantum states

- Size of $\mathcal{H}$ = Vol($2^{K+1} - 1$ sphere)
- Vol(ϵ ball) = Vol($2^{K+1} - 2$ sphere)

$$\frac{Vol(Physical\ states)}{Vol(States)} \in \mathcal{O}(K^K \epsilon^{2^k})$$

Summary

- Folklore (in physics): High frequency fluctuations don't affect the low energy physics.
  - We have demonstrated this rigorously for the class of k-local Hamiltonians.
- Corollary: Using randomness, it is possible to efficiently simulate k-local bounded Hamiltonians with arbitrary time-dependence.
  - Previous techniques had complexity scaling with the inverse fluctuation time-scale of the system.
- Folklore (in this community): Most states in Hilbert space are inaccessible.
  - Proved this for dynamics generated by arbitrary k-local Hamiltonian.
  - Physically well motivated.
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