Bringing order through disorder: Localization in the toric code

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Motivation

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- The toric code is a **quantum memory**, protected by **topology** and a **gap**
- In experiments, memories are subject to stray **magnetic fields**
- Their effect on the gap and on topological order have been well studied (Bravyi and Hastings 2010; J. Vidal et al 2008; Tsomokos et al 2010)
- Here we study their **dynamic effects** on excited states (Kay 2008; Pastawski, Kay, Schuch, Cirac 2009)
- Anyonic errors are propagated via **quantum walks**
- Quantum **memory** is **destroyed in linear time**
- *Can **disorder** be used to protect the stored information through **localization**?*

Overview

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- Toric code
  - Encoding, errors and anyons
  - Hamiltonian and protection

- Magnetic fields
  - Effects on the toric code
  - Quantum walks

- Disorder and localization
  - Random couplings
  - Anderson localization
  - Error suppression

The toric code

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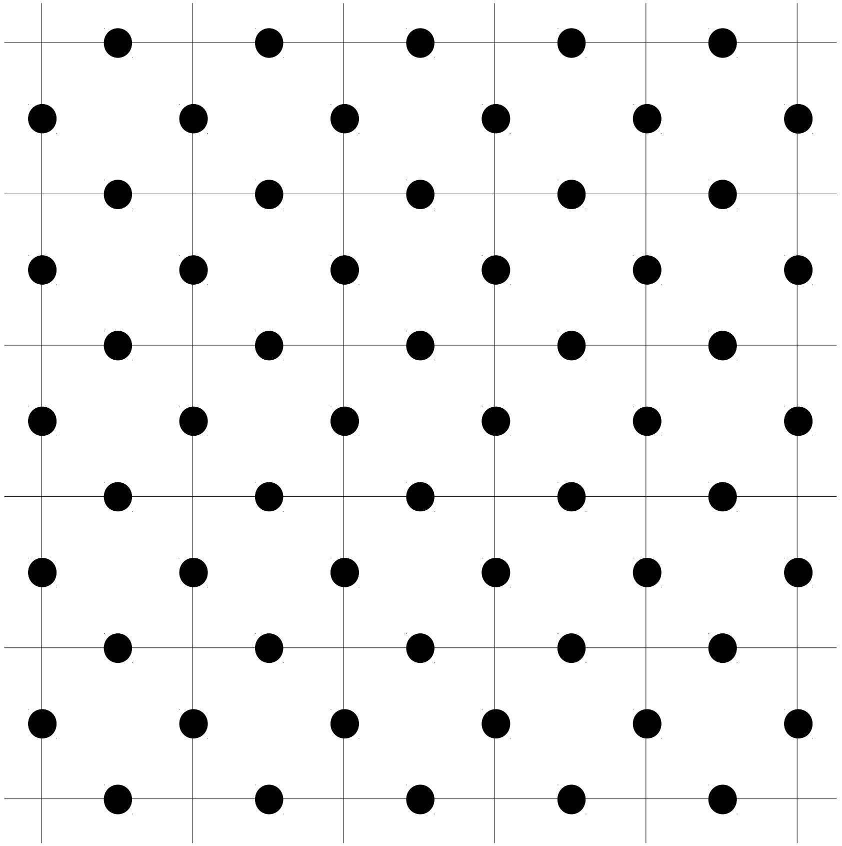
- Proposed by Kitaev (1997)
- Stabilizer code
- Defined on 2D lattice
- Spin-1/2 on edges
- Lattice wrapped around torus
  (other surfaces may also be used)



The toric code

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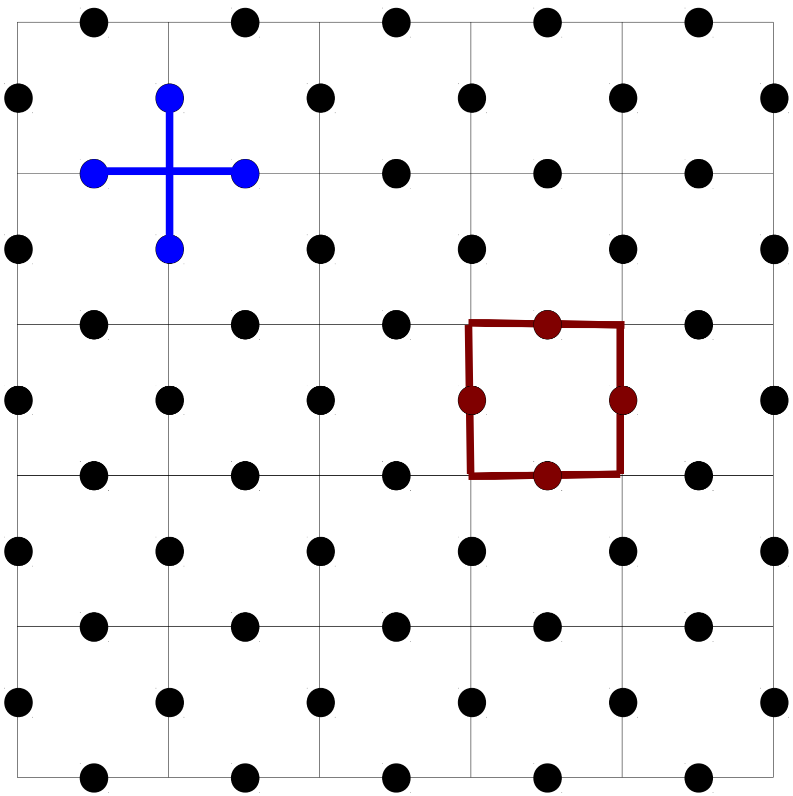
- Stabilizers defined on spins around each plaquette and vertex

$$A_v = \sigma_i^x \sigma_j^x \sigma_k^x \sigma_l^x ,$$

$$B_p = \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$



- Quantum information stored in stabilizer space

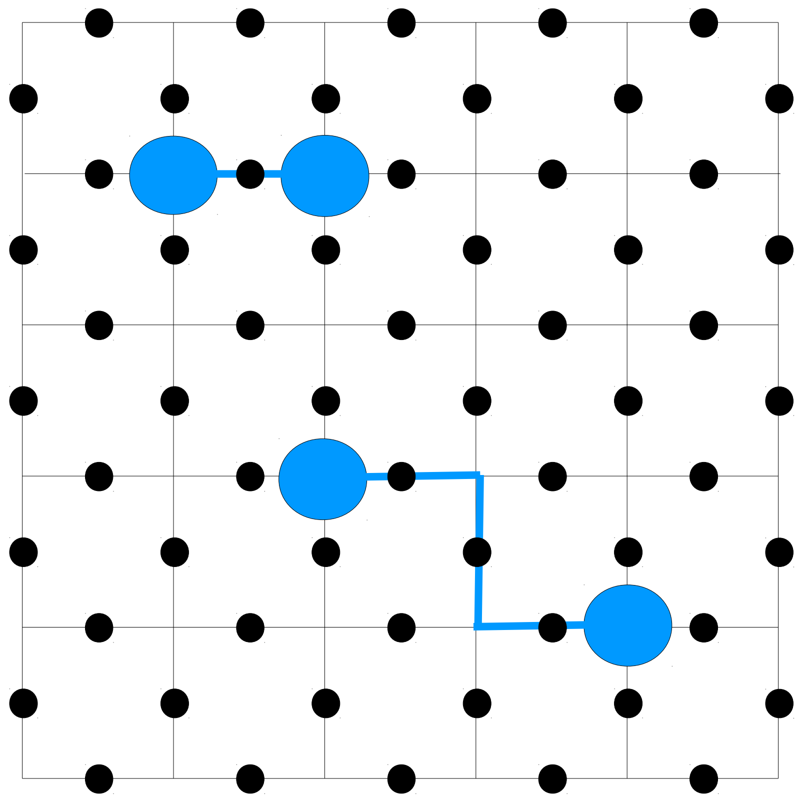
$$A_v |\psi\rangle = |\psi\rangle \; \forall \; v$$

$$B_v |\psi\rangle = |\psi\rangle \; \forall \; p$$

- Four dimensional Hilbert space: two logical qubits

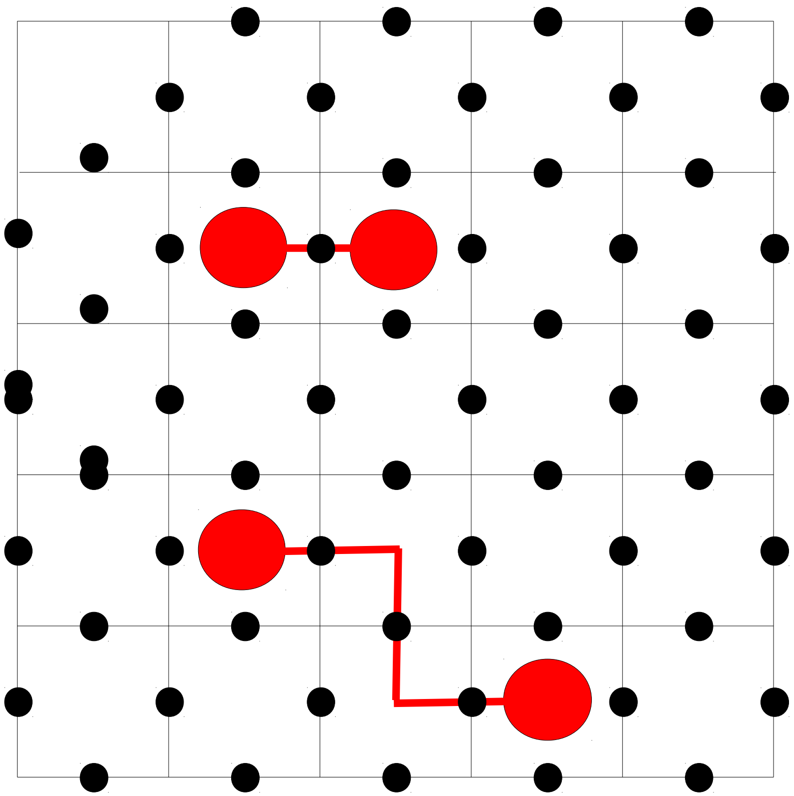
The toric code

- Local spin errors move state out of stabilizer space
- Stabilizers can be measured to determine whether errors have occurred
- Best means to correct errors can be determined and performed
- Single spin errors affect pairs of neighbouring stabilizers
- Can be interpreted as pair creation of quasiparticles
- $A_v|\psi\rangle = -|\psi\rangle$ implies an e anyon on v
- Created and moved by σ_i^z spin errors



The toric code

- $B_p|\psi\rangle = -|\psi\rangle$ implies an m anyon on p
- Created and moved by σ_i^x operations
- The anyons have mutual anyonic statistics, but this will not prove important



The toric code

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- **Logical operations** correspond to **moving anyons** around the torus in topologically non-trivial loops

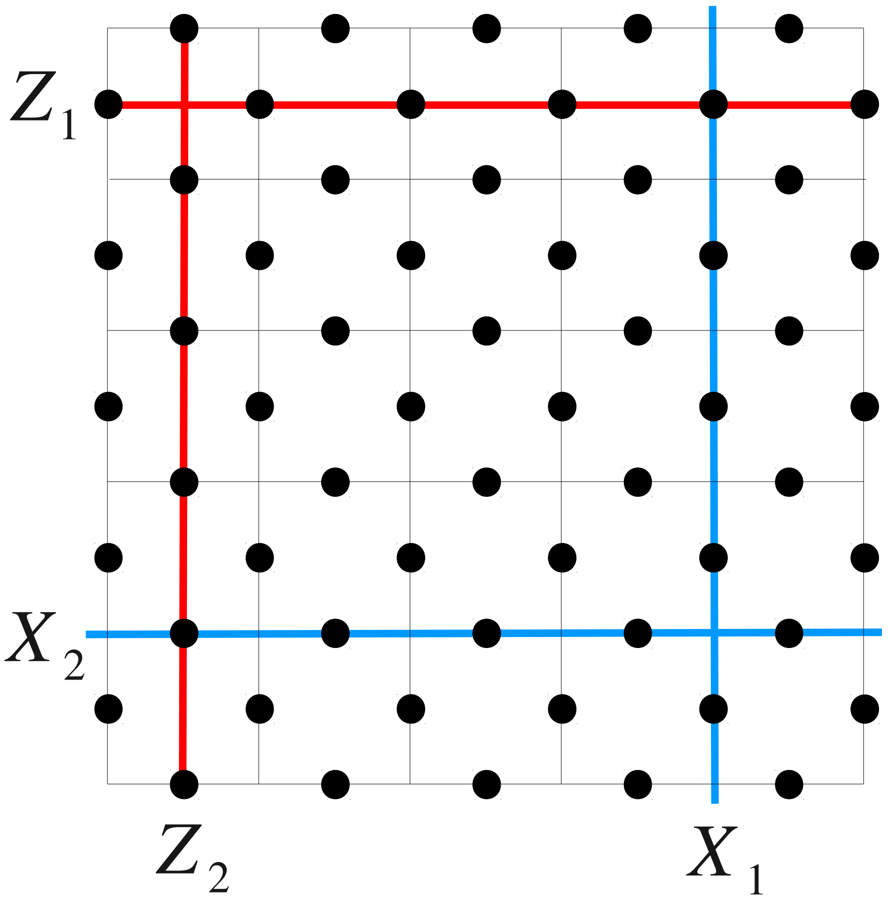
- Trivial loops have no effect on logical qubits – equivalent to stabilizers

- **Error** correction attempts to annihilate anyons without creating **non-trivial loops**

- Error correction successful when density of anyons is less than a **critical value**

$$\rho_c \approx 0.31$$

(Dennis, Kitaev, Landahl, Preskill, 2002)



The toric code

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- Quasilocal stabilizers mean **Hamiltonian** can be implemented

$$H_{TC} = -J\sum_v A_v - J\sum_p B_p$$

- **Degenerate ground state** corresponds to stabilizer space

- Encoded information protected by **energy gap**

- **Gap stable** against local perturbations (this morning's talk), but information vulnerable to dynamic effects (Pastawski)

Magnetic fields and the toric code

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- Consider the perturbation

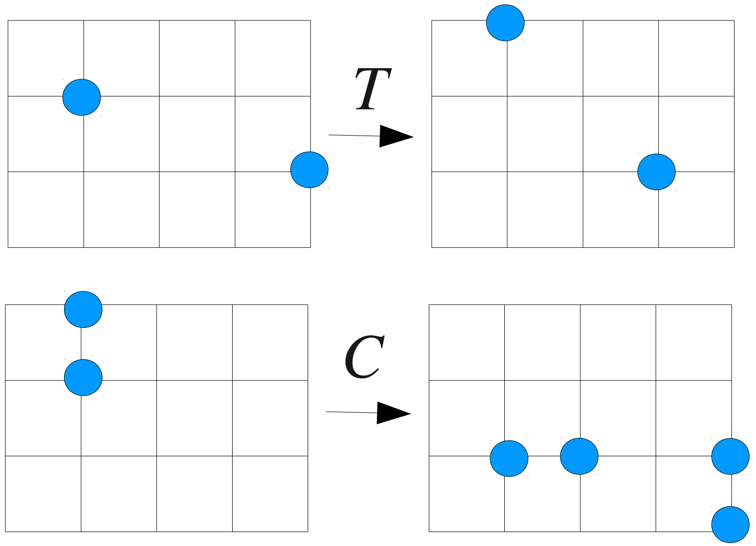
$$H_{TC} = -J\sum_v A_v - J\sum_p B_p + h\sum_i \sigma_i^z$$

$$\sum_i \sigma_i^z = T + C$$

$$T = \sum_n P_n \left(\sum_i \sigma_i^z\right) P_n$$

$$C = \sum_{n \neq m} P_n \left(\sum_i \sigma_i^z\right) P_m$$

- P_n is the projector onto the space of states with n vertex anyons
- T moves e anyons
- C creates and annihilates them



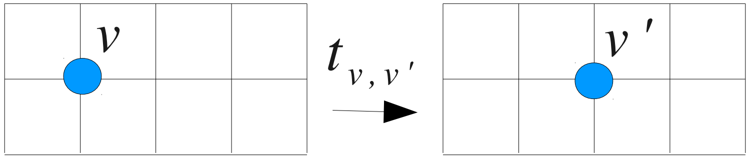
2D anyonic quantum walks

- For $J \gg h$ the effect of C is perturbatively suppressed

- The Hamiltonian then moves any anyons present in a continuous time quantum walk

$$H = \sum_{v,v'} M_{v,v'} t_{v,v'} + U \sum_{v} n_v (n_v - 1)$$

$$M_{v,v'} = J\,\delta_{v,v'} + h\,\delta_{\langle v,v' \rangle}$$



- Such walks spread quickly, causing logical errors in a time **linear** with L

- Critical density of anyons is zero in the presence of the field. No errors can be tolerated.

- **Anyonis statistics** do not have a significant effect (Pachos et al, 2009)

Disorder in Couplings

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- Can we suppress the effect of the **magnetic field** and regain finite **critical density**?

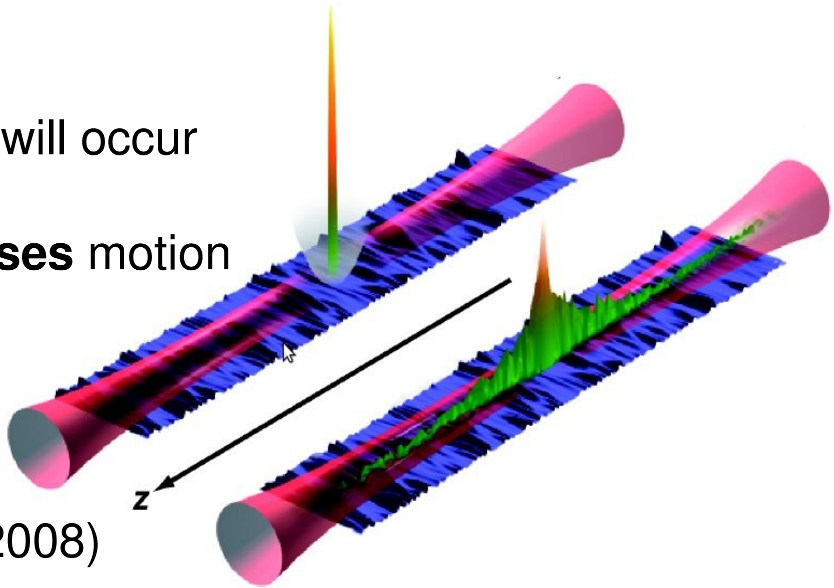
- Consider disorder in the toric code Hamiltonian

$$H_{TC} = -\sum_v J_v A_v - \sum_p J_p B_p \qquad M_{v,v'} = J_v \delta_{v,v'} + h\,\delta_{\langle v,v'\rangle}$$

- J_v randomly vary from vertex to vertex

- Theory suggests that **Anderson localization** will occur

- Random interference **exponentially suppresses** motion

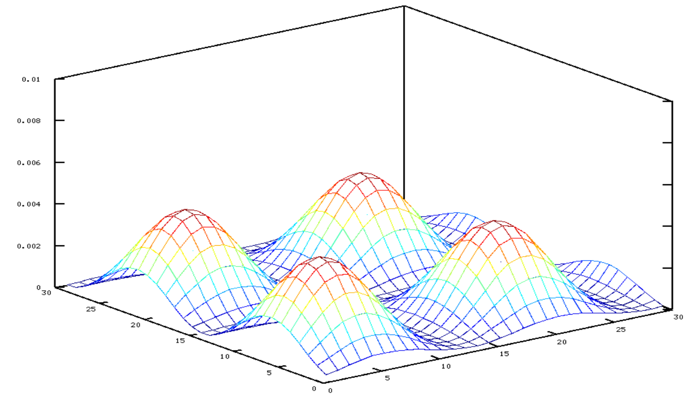


(Aspect et. al, 2008)

Disorder in Couplings

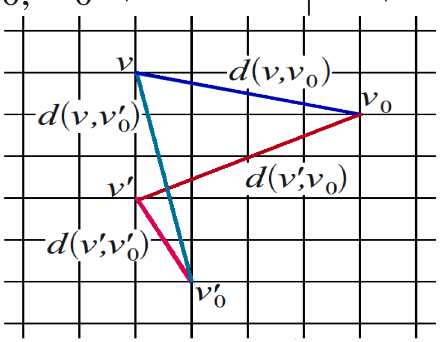
- Consider sparsely distributed anyon pairs. Only two walker Hamiltonians need be considered

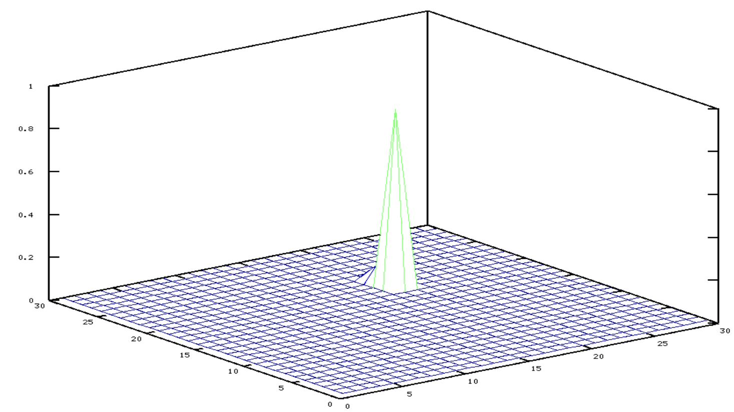
- Bound can be placed on eigenstates



$$\left|\left\langle v\, v' \middle| E_{v_0 v_0'} \right\rangle\right| < \exp\left[\frac{-d(v, v'; v_{0,} v_0')}{2l_{v_{0,} v_0'}}\right]$$

$$d(v, v'; v_{0,} v_0') = min\left| d(v, v_0) + d(v', v_0'), d(v, v_0') + d(v', v_0) \right|$$





- Hamiltonian localization length l defined as maximum of all $l_{v_{0,} v_0'}$

Disorder in Couplings

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- Localized eigenstates prevent the walkers moving freely

- **Motion** of the walker is **exponentially suppressed**

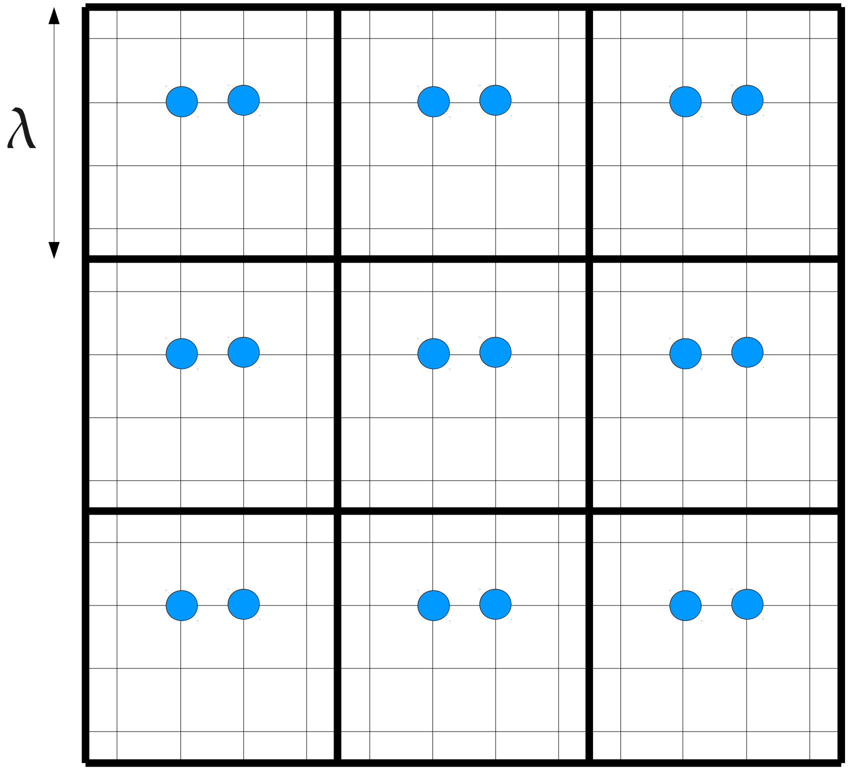
$$P(d,t) < L^8 e^{-d/l} \approx (21)^8 e^{-d/l}$$

- **Anyons are bound** to an area of radius ~*l* around their starting position at all times t.

- This allows a **finite anyon density** to be tolerable, even in the presence of the field

Disorder and Error Suppression

- For an approximation of the critical density, consider a coarse grained lattice



- Anyon pair in each box
- Errors occur when anyons leave their boxes
- Due to localization

$$p < 2^8 l^9 e^{-\lambda / l}$$

- Errors correctable when

$$p < p_c, \quad p_c \approx 0.11$$

(Dennis, et al)

$$\lambda > l \ln\left(2^8 l^9 / p_c\right)$$

hence

$$\rho_c < [l \ln\left(2^8 l^9 / p_c\right)]^{-2}$$ and so not zero

Disorder and Error Suppression

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- Localization lengths in 2D can be very large

- If too large, it may not be realistic to build codes big enough to benefit from localization

- Critical anyon density, though non-zero, will then become impractically small

- It's therefore important to **determine the typical values of *l*** for disorder we would expect to be inherent in realizations of the toric code

- Consider Cauchy distribution around average value J. Disorder parametrized by width γ

Disorder and Error Suppression

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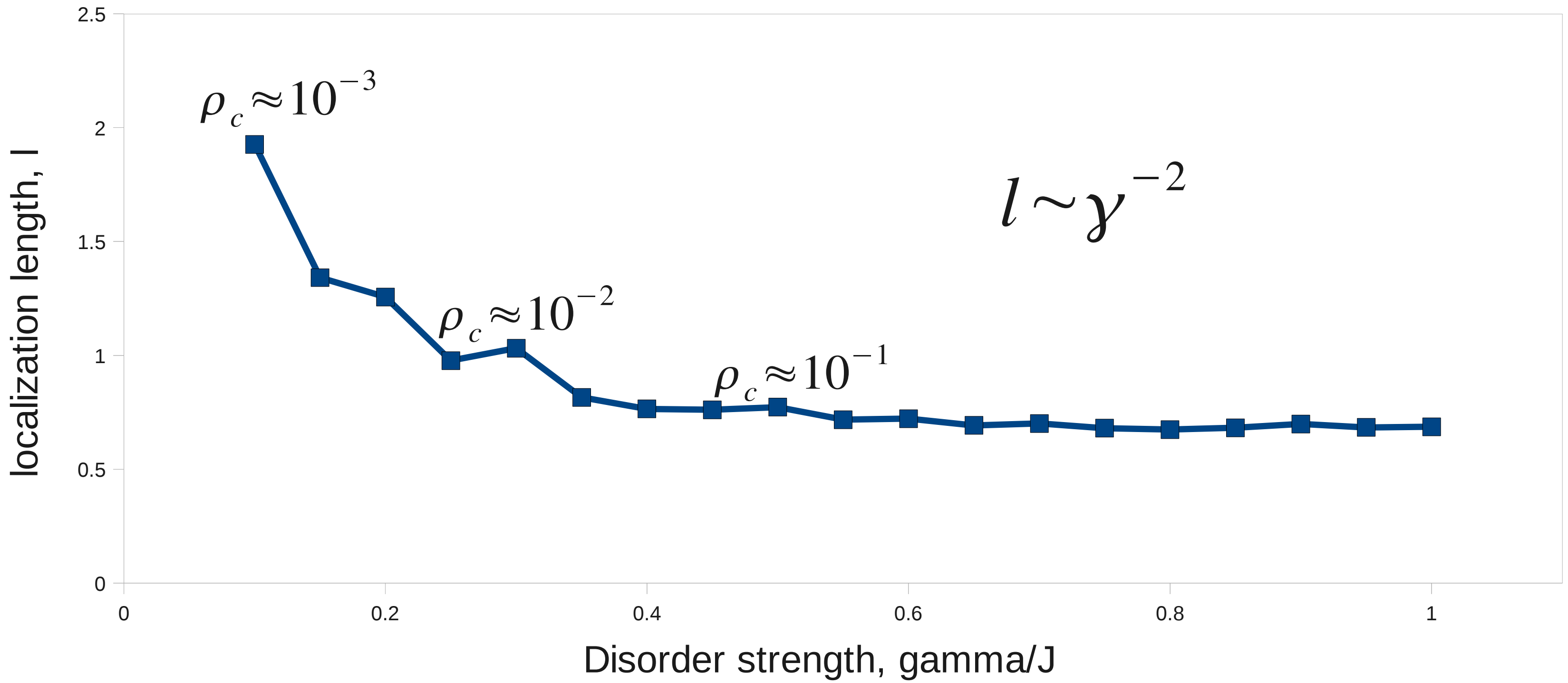
- Other disorder strengths were also considered



- Length decreases for increased disorder

- May be worth purposefully including disorder in the toric code to enhance localization effect

Conclusions

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- Magnetic fields are fatal for the toric code, inducing quantum walks
- This destroys memory in linear time, and sets the critical anyon density to zero
- Disorder inherent in the J's will cause Anderson localization, exponentially suppressing anyon motion and allowing the critical anyon density to be finite
- Other sources of disorder have also been considered. Using random graphs causes linear lifetime to become polynomial.
- Random graphs also increase lifetime against thermal errors, which induce classical random walks of anyons
- Disorder is powerful tool to suppress errors in topological models
- Ultimate goal: thermally stable memories. Could disorder help toward this goal?

The End

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- Thanks for your attention

Random Lattices

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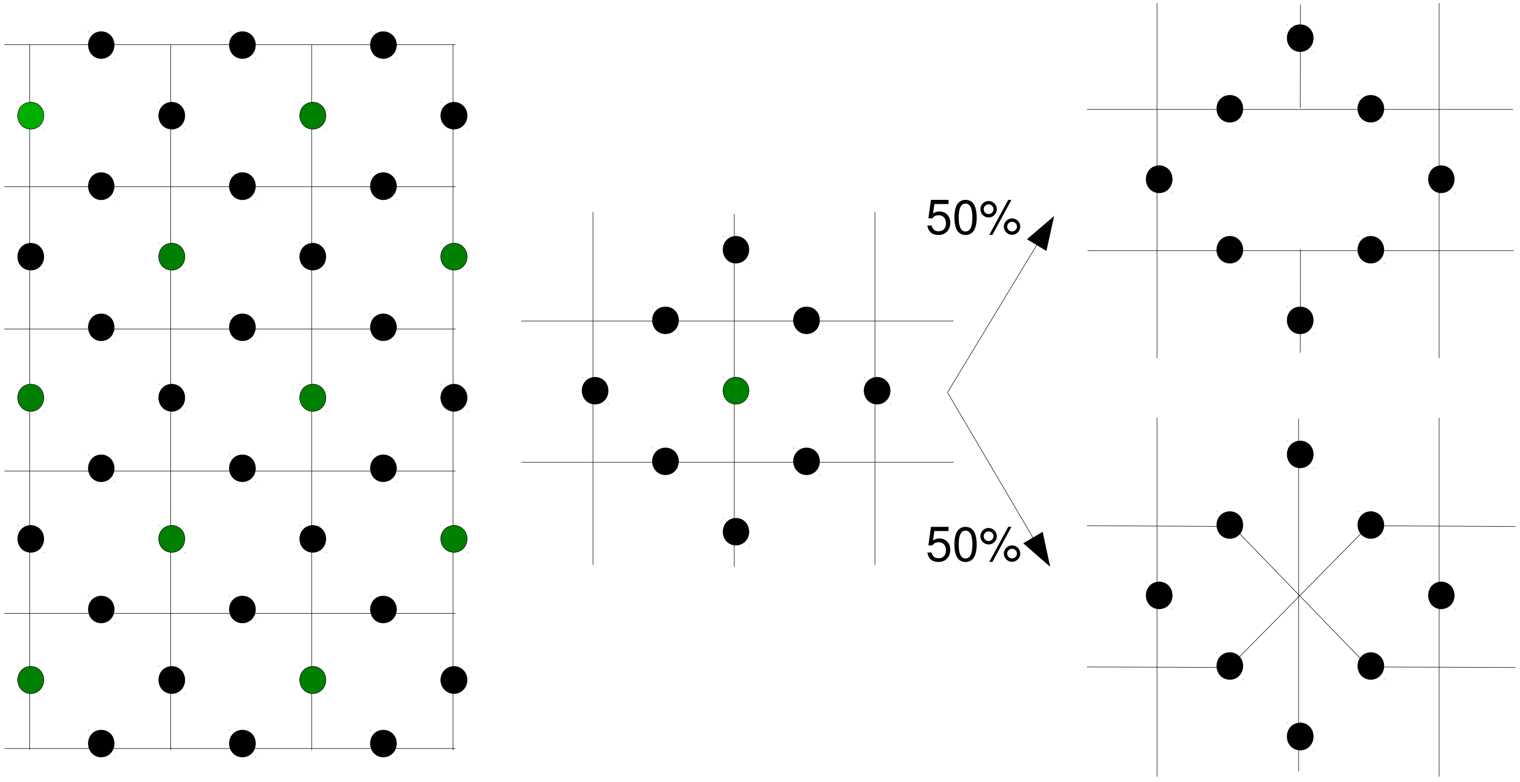
- We consider **random lattices**
- These are designed such that
  - -Number of spins per plaquette and vertex remains small
  - -Symmetry is maintained between e and m anyons



Random Lattices

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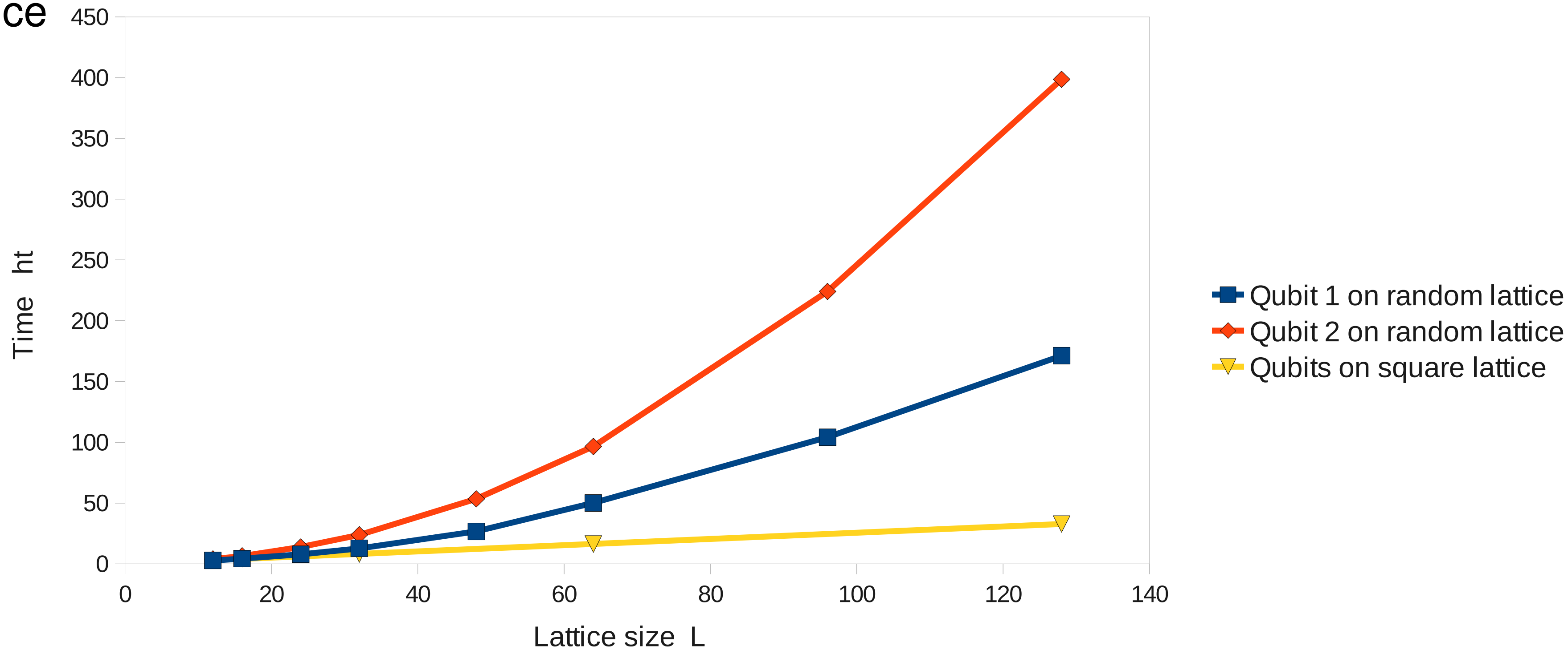
- The speed at which errors build up can be seen from the time taken until the error probability becomes p=0.1
- This increases linearly with L on the square lattice, but polynomially for the random lattice



- Lifetime is greatly increased by disorder

- Note also that only ¾ of the spins are used

Thermal Errors

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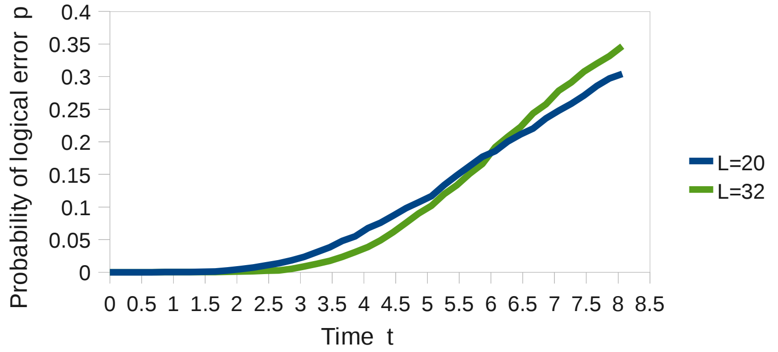
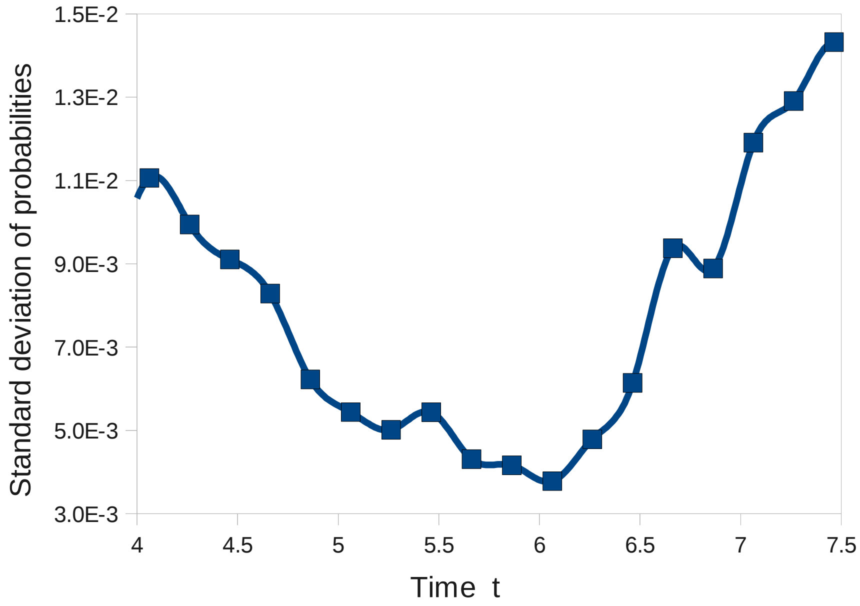
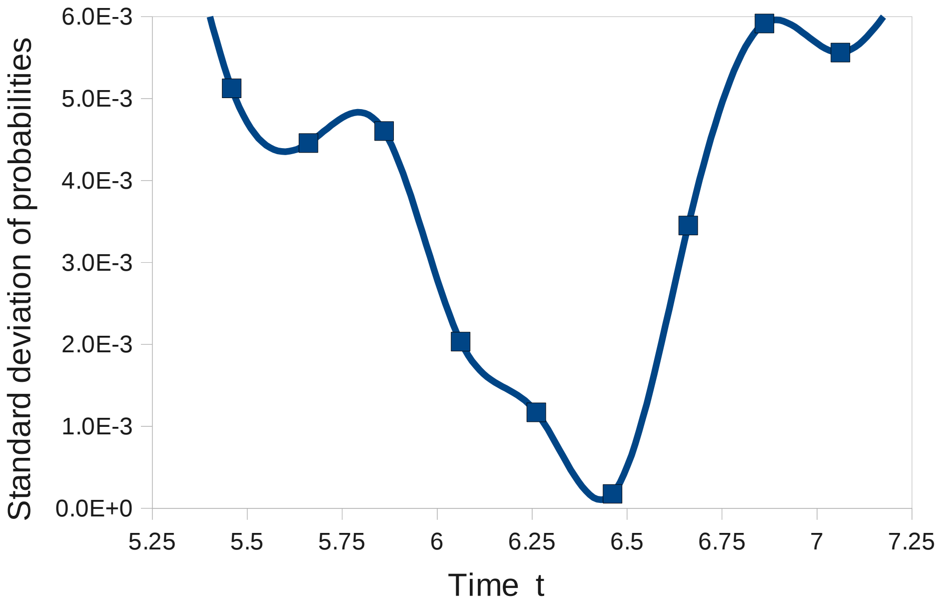
- Thermal errors induce classical random walks of anyons
- Anderson localization not possible
- However, random graphs may still have effect
- Increase of the critical time is found







Disorder and Error Suppression

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- Two walker Hamiltonian was diagonalized for

$$\gamma = J/10 \quad h = J/100$$

- Probability distribution derived from each eigenstate

- Localization length of the eigenstate taken to be s.d. of distribution

- Localization length of Hamiltonian is maximum of all these