Information propagation for interacting particle systems

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Abstract

We show that excitations of interacting quantum particles in lattice models – and thus information in these systems – always propagate with a finite speed of sound. Our argument is simple yet general and shows that by focusing on the physically relevant observables one can typically expect a bounded speed of information propagation. It applies equally to quantum spins, bosons such as in the Bose-Hubbard model, fermions, anyons, and general mixtures thereof, on arbitrary lattices of any dimension. It also pertains to dissipative dynamics on the lattice, and generalizes to the continuum for quantum fields. Our result can be seen as a meaningful analogue of the Lieb-Robinson bound for strongly correlated models.

[The technical version, which also includes references, has been submitted as an attachment.]

How fast can information propagate through a system of interacting particles? The obvious answer seems: No faster than the speed of light. While certainly correct, this is not the answer one is usually looking for. For instance, in a classical solid, liquid, or gas, perturbations rather propagate at the speed of sound, which is determined by the way the particles in the system locally interact with each other, without any reference to relativistic effects. We would like to understand whether a similar “speed of sound” exists for interacting quantum systems, limiting the propagation speed of localized excitations, i.e., (quasi-)particles. For interacting quantum spin systems, such a maximal velocity, known as the Lieb-Robinson bound, has indeed been shown. While it seems appealing that there should always be such a bound, systems of interacting bosons can show counterintuitive effects, in particular since the interpretation of excitations in terms of particles is no longer fully justified; in fact, an example of a non-relativistic system where bosons are steadily accelerated through a lattice has recently been constructed. This example suggests the disturbing possibility that our intuition about the propagation of information in quantum systems is wrong, and only relativistic quantum theory can provide a proper speed limit.

There are many important reasons, both theoretical and experimental, to investigate information propagation bounds in interacting particle systems. It turns out that such bounds lead directly to important, general results concerning the clustering of correlations in equilibrium states. Lieb-Robinson bounds also facilitate the simulatability of strongly interacting quantum systems – the mere existence of a Lieb-Robinson bound for a quantum system can be used to develop general, efficient, numerical procedures to simulate the dynamics of lattice models. From a more practical perspective, new experiments allow one to explore the non-equilibrium dynamics of ultracold strongly correlated quantum particles – bosonic, fermionic, or mixtures thereof – in optical lattices with unprecedented control. In such experiments, it is important to understand how the particles move: For example, when studying instances of anomalous expansion, it is far from clear a priori whether it is possible to identify a meaningful speed of sound at all.

The original Lieb-Robinson bound already applies in a very general setting, namely, to any low-dimensional quantum spin system, and to any fermionic system confined to a lattice. It is therefore tempting to extend the original argument to other settings, in particular, to systems of interacting bosons; unfortunately, all attempts to do so have run into insuperable difficulties for systems with nonlinear interactions, including the Bose-Hubbard model. The reason for the failure of the original Lieb-Robinson argument is fundamentally connected to the unboundedness of the creation operator for bosons: The Lieb-Robinson velocity depends on the norm of the interaction, which is unbounded for e.g. bosons hopping on a lattice, and examples of bosonic systems where particles steadily accelerate have been constructed.

In our work, we show how these difficulties can be overcome by considering the right question concerning the propagation of information. Our approach allows us to determine Lieb-Robinson type bounds for the maximal speed at which information can propagate through systems of interacting particles in an extremely general scenario: In particular, our approach applies to systems of interacting bosons, as well as to fermions, spins, anyons, or mixtures thereof, both on lattices and in the continuum. Moreover, it can also be applied beyond Hamiltonian evolution, such as to systems evolving under some local dissipative dynamics.

The type of system we consider in our work is exemplified by the Bose-Hubbard model, a model of bosonic particles hopping on an arbitrary lattice G of any finite dimension and interacting via an on-site repulsion,

$$\hat{H}_{\mathrm{BH}} = -\tau \sum_{\langle j,k \rangle} (\hat{b}_j^\dagger \hat{b}_k + \mathrm{h.c.}) + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) \ .$$

Here, the first summation is over neighboring sites on the lattice, $\hat{b}_j$ is the boson annihilation operator for site j, and $\hat{n}_j = \hat{b}_j^\dagger \hat{b}_j$ is the number operator. Our arguments generalize directly to models consisting of bosons, fermions, anyons, or mixtures thereof, with an arbitrary (even non-local) attraction or repulsion term which depends only on the local particle numbers.

The scenario we consider is that of an initially empty lattice, where at time $t = 0$ particles are placed in an arbitrary quantum state in some region R of the lattice (R can well cover most of the lattice). What we want to know is how fast these particles propagate into the initially empty region of the lattice: We want to find a velocity v such that at any site j which has lattice distance l to region R, we have that for all times $t < l/v$ no particles (and thus no signal whatsoever) have travelled from R to j, up to a correction exponentially small in $l - vt$.

The main insight that allows us to derive these bounds is that, in order to understand the propagation of particles, it is *not* necessary to consider the evolution of the initial state as a whole – this would indeed be an impossible task, given the superexponential dimension of the underlying Hilbert space. Rather, it is sufficient to focus on a set of relevant observables. In our case, these are the local particle densities

$$\alpha_j(t) := \mathrm{tr}[\hat{n}_j \rho(t)]$$

at site j at time t. As we can show, the time derivatives of the α_j form a closed system of differential inequalities. Thus, the worst-case solution of this system of inequalities that satisfies the initial conditions yields a bound on how fast particles, and thus information, propagate in the lattice. Indeed, it allows us to infer a bound v on the speed of information propagation (which only depends on the hopping rate τ and the structure of the underlying graph), up to an exponentially small correction: $\alpha_j(t) \leq \text{const.} \times \exp[vt - l]$, where l is the distance of j from R. This, somewhat surprisingly, demonstrates that the propagation of particles can be understood by considering the evolution of a restricted set of observables, rather than the evolution of the full quantum state of the system.

Our results on the propagation of particles as detected by $\alpha_j(t)$ can be readily generalized to the propagation of information as detected by any kind of local observable, since the measured signal can always be bounded by the number of particles which have already propagated. We also discuss how to generalize our results to the case where one has several species of bosons, fermions, anyons, or Bose-Fermi mixtures as is the case in experiments. Our techniques do not only apply to Hamiltonian evolution, but also to a range of dissipative systems, and can be generalized to continuous theories.

The technical version, which also includes references, has been submitted as an attachment.